Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with ** = 45 minutes and ** = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

A. 0.3875

B. 0.2676

C. 0.5

D. 0.6987

Answer :

X = 60 , Mean = 45+10 = 55 , Std. Deviation = 8

from scipy import stats

round(1-stats.norm.cdf(60,loc=55,scale=8),5) Output : 0.26599

Option B is the correct answer.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean ** = 38 and Standard deviation ** =6. For each statement below, please specify True/False. If false, briefly explain why.
   1. More employees at the processing center are older than 44 than between 38 and 44.

Answer :

False ,

mean = 38 & SD = 6

means that, most of the ages are lying between 32 and 44

#Z-score for 44

from scipy import stats

round(1-stats.norm.cdf(44,loc=38,scale=6),4)

Output : 0.1587

i.e. 63 employees out of 400

#Z-score between 38 and 44 from scipy import stats

round(stats.norm.cdf(44,loc=38,scale=6) - stats.norm.cdf(38,loc=38,scale=6),4)

Output : 0.3413

i.e. 137 employees out of 400 therefore , 137 > 63 hence given condition is false.

* 1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer :

True ,

from scipy import stats round(stats.norm.cdf(30,loc=38,scale=6),4)

Output : 0.0912

i.e. 36 employees out of 400 hence given condition is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer :

We know that ,

if *X 1* ~ *N*(μ, σ2) and *X2* ~ *N*(μ, σ2) are two independent random variables then , X1 + X2 ~ *N*(μ + μ , σ2 + σ2 )

Similarly if Z = aX1 + bX2 , where X and Y are as defined above, i.e Z is linear combination of X1 and X2 , then Z ~ N(aµ + bµ, a2 σ2 + b2 σ2 ).

Therefore from the question ,

2X1 ~ N(2 µ , 4 σ2 ) & X1+X2 ~ N(2 µ, 2σ2 )

2X1 - (X1+X2) = N( 0,2σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9

B. 80.2, 119.8

C. 22, 78

D. 48.5, 151.5

E. 90.1, 109.9

Answer :

Since a and b are symmetric about mean, It is two tailed test ,

Hence , for 99% , we need to add 0.5% on either side = 0.99+0.005 = 0.995

Z value of 0.005 is ,

from scipy import stats stats.norm.ppf(0.005)

Z value of 0.005 = -2.57

Now, Z value of 0.995 is ,

from scipy import stats stats.norm.ppf(0.995)

Z value of 0.995 = 2.57

Hence ,

x–Mean

Z =

SD

Hence ,

therefore ,

x = SD \* Z + Mean x = 20\*Z + 100

a = (20 \* (-2.57)) + 100

a = 48.5

b = (20\* 2.57) + 100

b = 151.5

Option D. (48.5, 151.5 ) is correct answer.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Answer :

import numpy as np from scipy import stats mean = 5+7

print('Mean Profit is Rs', mean\*45,'Million') sd = np.sqrt((9)+(16))

print('Standard Deviation is Rs', sd\*45, 'Million')

print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

Output : Mean Profit is Rs 540 Million

Standard Deviation is Rs 225.0 Million

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

* 1. Specify the 5th percentile of profit (in Rupees) for the company Answer :

we know that , Z value for 5th percentile is = -1.645

X = SD\*Z + Mean is ,

X= 540+(-1.645)\*(225)

print('5th percentile of profit is',round(X),'(in Million Rupees)')

Output : 5th percentile of profit is 170 (in Million Rupees)

* 1. Which of the two divisions has a larger probability of making a loss in a given year?

Answer :

Making loss , i.e X<0 Division 1 :

stats.norm.cdf(0,5,3)

Output : 0.04779035

Division 2 :

stats.norm.cdf(0,7,4)

Output : 0.04005915

Hence ,

Division 2 will face more loss.